Calculus Warm Up #2- 5

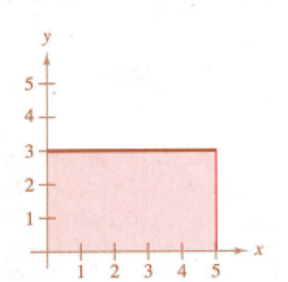
Estimate the area of the region bounded by f(x) and the x-axis on [4, 6] using 4 equal subintervals. Compare right and left endpoint areas.

$$f(x) = \frac{1}{x - 2}$$

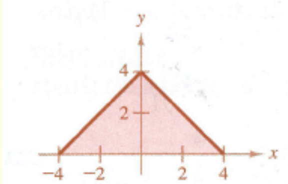
HW Questions: p. 258

In Exercises 1–10, set up a definite integral that yields the area of the given region. (Do not evaluate the integral.)

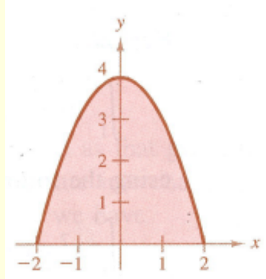
1. $f(x) = 3$



3. $f(x) = 4 - |x|$

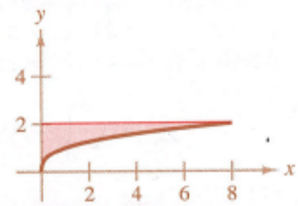


5. $f(x) = 4 - x^2$

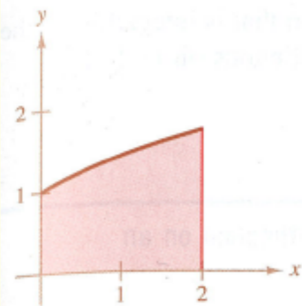


In Exercises 1–10, set up a definite integral that yields the area of the given region. (Do not evaluate the integral.)

7. $g(y) = y^3$



9. $f(x) = \sqrt{x + 1}$



In Exercises 11–20, sketch the region whose area is indicated by the given definite integral. Then use a geometric formula to evaluate the integral.

11. $\int_0^3 4\,dx$

13. $\int_0^4 x\,dx$

15. $\int_0^2 (2x + 5)\,dx$

17. $\int_{-1}^1 (1 - |x|)\,dx$

19. $\int_{-3}^3 \sqrt{9 - x^2}\,dx$

21. Given $\int_0^5 f(x)\,dx = 10$ and $\int_5^7 f(x)\,dx = 3$, find the following.

(a) $\displaystyle\int_0^7 f(x)\,dx$ (b) $\displaystyle\int_5^0 f(x)\,dx$

(c) $\displaystyle\int_5^5 f(x)\,dx$ (d) $\displaystyle\int_0^5 3f(x)\,dx$

23. Given $\int_2^6 f(x)\,dx = 10$ and $\int_2^6 g(x)\,dx = -2$, find the following.

(a) $\displaystyle\int_2^6 [f(x) + g(x)]\,dx$ (b) $\displaystyle\int_2^6 [g(x) - f(x)]\,dx$

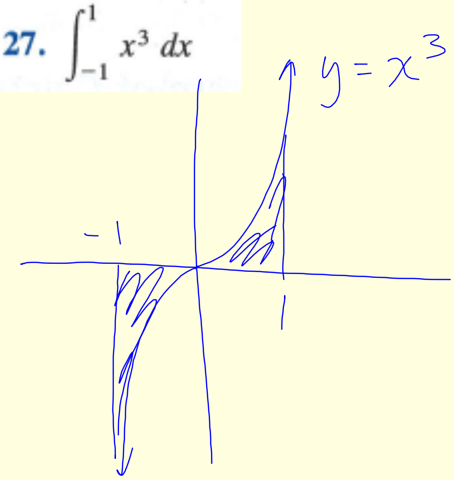
(c) $\displaystyle\int_2^6 2g(x)\,dx$ (d) $\displaystyle\int_2^6 3f(x)\,dx$

In Exercises 25–30, evaluate the definite integral by the limit definition.

25. $\displaystyle\int_4^{10} 6\,dx$

27. $\displaystyle\int_{-1}^{1} x^3\,dx$

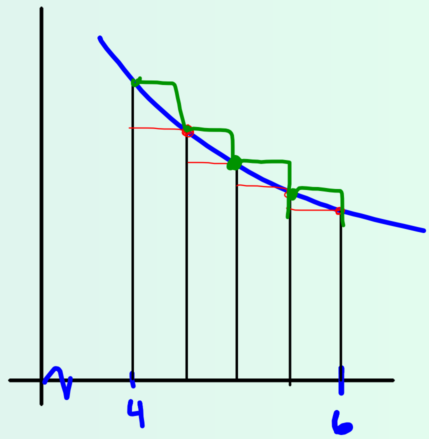
29. $\displaystyle\int_1^2 (x^2 + 1)\,dx$



From the warm up:

$0.635 < A < 0.760$

$f(x) = \dfrac{1}{x-2}$, $[4, 6]$



Right endpoints:

undestimation

Left endpoints:

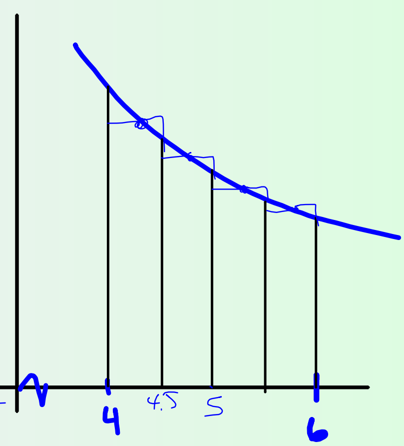
overestimation

What about using midpoints?

From the warm up:

$0.635 < A < 0.760$

$f(x) = \dfrac{1}{x-2}$, $[4, 6]$



Midpoints:

4.25, 4.75, 5.25, 5.75

$A = \frac{1}{2}\left[f(4.25) + f(4.75) + f(5.25) + f(5.75)\right]$

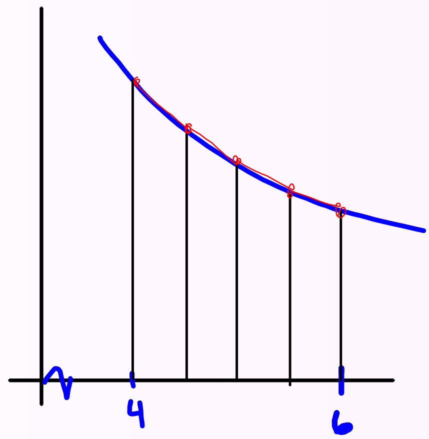
$= \frac{1}{2}\left(\frac{1}{2.25} + \frac{1}{2.75} + \frac{1}{3.25} + \frac{1}{3.75}\right)$

≈ 0.691

From the warm up:

$0.635 < A < 0.760$

$f(x) = \dfrac{1}{x - 2}$, [4, 6]



What if we used trapezoids?

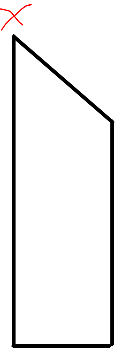
Area of a Trapezoid

$A = \frac{1}{2}h(b_1 + b_2)$

Δx

$b_1 = f(x_{i-1})$

b_1

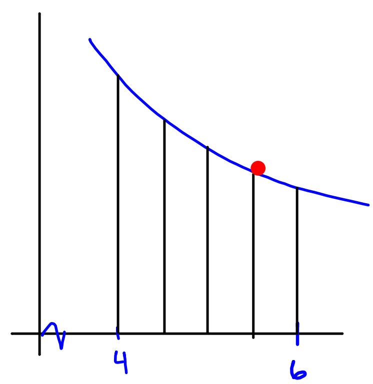


$b_2 = f(x_i)$

Δx

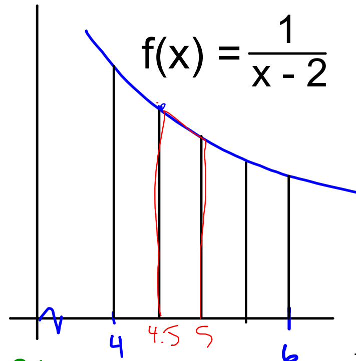
Right endpoint, x_i

Left endpoint, x_{i-1}



Area of a Trapezoid

$$A = \frac{1}{2}\Delta x\left(f(x_{i-1}) + f(x_i)\right)$$



$$A = \frac{1}{2}\cdot\frac{1}{2}\left[f(4) + \underbrace{f(4.5) + f(4.5)} + \underbrace{f(5) + f(5)} + \underbrace{f(5.5) + f(5.5)} + f(6)\right]$$

$$= \frac{1}{4}\left[\frac{1}{2} + 2\left(\frac{1}{2.5}\right) + 2\left(\frac{1}{3}\right) + 2\left(\frac{1}{3.5}\right) + \frac{1}{4}\right]$$

$$= 0.697$$

Area approximations

By right and left endpoints:

$0.635 < A < 0.760$

By midpoints: $A \approx 0.691$

By trapezoids: $A \approx 0.697$

Actual ? If the function is integrable, check on the calculator!

$\boxed{\text{MATH}}$ fnInt(1/(x-2), x, 4, 6)

$$A = \int_4^6 \frac{1}{x-2}\,dx \approx 0.693$$

HW: p. 287, # 3, 5, 9

(Just use trapezoids)

p. 250, # 57 & 58

(Midpoints)